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AN EFFECT OF A TRANSVERSE TEMPERATURE GRADIENT ON TURBULENT PIPE FLOW

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IT IS USUAL, when investigating the distribution of velocity and its relation to shear stress in non-isothermal turbulent flow through pipes or channels, either to neglect the influence of a temperature gradient or to consider only its effect on the regions near the heated boundaries. This is reasonable since the highest temperature gradients normally occur here; moreover fluid viscosity, which is usually strongly temperature dependent, plays a more important role in these regions than in the turbulent core.

In recent years nuclear engineers have had to deal with turbulent flow and heat transfer in ducts of various shapes, including annulii and channels containing bundles of rods. In such multiply-connected shapes, containing perhaps elements having roughened or extended surfaces, it is important to be able to isolate the contribution of an individual element to the overall heat transfer and pressure drop. A way of doing so involves locating the position of the "surface of zero shear" which surrounds the element in question [1]. In dealing with simple shapes it is assumed that zero shear occurs at the section where the mean velocity is a maximum, on the basis that the shear stress is proportional to $\epsilon_m \, \mathrm{d}\bar{U}/\mathrm{d}y$ where ϵ_m is the usual "eddy viscosity", $\bar{U}$ is the mean velocity and y is an appropriate co-ordinate normal to $\bar{U}$. With real fluids, however, and especially with gases, the existence of a transverse temperature gradient implies a transverse density gradient; the validity of the technique therefore needs investigation.

If a Reynolds-type substitution is made in the Navier–Stokes equations for steady compressible flow of a fluid having a constant viscosity, and time-averages are taken, one obtains:

$$\bar{\rho}\,\bar{U}_j \frac{\partial \bar{U}_i}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \mu \left[\frac{\partial^2 \bar{U}_i}{\partial x_j\, \partial x_j} + \frac{1}{3}\frac{\partial}{\partial x_i}\left(\frac{\partial \bar{U}_k}{\partial x_k}\right)\right]$$

$$-\frac{\partial}{\partial x_j}\left[\bar{\rho}\,\overline{u_i u_j} + \overline{\tilde{\rho}\, u_j}\,\bar{U}_i + \overline{\tilde{\rho}\, u_i}\,\bar{U}_j + \overline{\tilde{\rho}\, u_i u_j}\right] \quad (1)$$

where $\bar{U}_i$, $\bar{\rho}$, u_i and $\tilde{\rho}$ are the steady and fluctuating components of velocity and density respectively and $\bar{p}$ is the steady component of the static pressure.

We assume that the fluid behaves as a perfect gas, having an equation of state

$$\bar{p} + \tilde{p} = R(\bar{\rho} + \tilde{\rho})(\bar{T} + \theta) \quad (2)$$

where $\bar{T}$ and θ are the steady and fluctuating components of absolute temperature. We further assume that $\tilde{p}$ is negligible, i.e. that density changes resulting from an imposed temperature gradient occur at constant pressure. Then equation (2) yields

$$\bar{\rho}\theta + \tilde{\rho}T = \overline{\tilde{\rho}\theta} - \tilde{\rho}\theta \quad (3)$$

In most practical situations $T \geqslant 300°\mathrm{K}$. Accurate data on θ in the core of turbulent pipe flow do not appear to exist. We should expect, however, by analogy with corresponding velocity fluctuations that, with the usual magnitude of temperature differences in engineering equipment, θ would be of the order of 10°C. We therefore assume that $\tilde{\rho}\theta \ll \tilde{\rho}\bar{T}$. Since $\overline{\tilde{\rho}\theta} \leqslant \tilde{\rho}\theta$ we neglect the right-hand side of equation (3) and write

$$\bar{\rho}\theta + \tilde{\rho}\bar{T} = 0 \quad (4)$$

Using equation (4), the last term, equation (1) can be transformed into

$$-\frac{\bar{p}}{R}\frac{\partial}{\partial x_j}\left[\frac{\overline{u_i u_j}}{\bar{T}} - \frac{\overline{\theta u_j}}{\bar{T}^2}\bar{U}_i - \frac{\overline{\theta u_i}}{\bar{T}^2}\bar{U}_j - \frac{\overline{\theta u_i u_j}}{\bar{T}^2}\right] \quad (5)$$

Consider now the case of steady flow in the Ox direction between infinite parallel plates perpendicular to Oy. Assume that a steady temperature gradient exists in the Oy direction and that $\mathrm{d}\bar{p}/\mathrm{d}x$ is a constant which is small enough to permit the neglect of density changes in the Ox direction. Then, associating $\bar{U}$, $\bar{V}$, $\bar{W}$, u, v, w with the steady and fluctuating velocity components in the x, y, z direction respectively we have

$$\bar{U} = \bar{U}(y), \quad \bar{V} = \bar{W} = 0, \quad \bar{T} = \bar{T}(y)$$

and all the averages of fluctuations are functions of y only. If we neglect the last term in equation (5) (the triple correlation), the i-component of equation (1) becomes

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$$\frac{d\bar{p}}{dx} = \mu\frac{d^2\bar{U}}{dy^2} - \frac{\bar{p}}{R}\frac{d}{dy}\left[\frac{\overline{uv}}{\bar{T}} - \frac{\overline{\theta v}}{\bar{T}^2}\bar{U}\right] = \mu\frac{d^2\bar{U}}{dy^2}$$

$$+ \frac{\bar{p}}{R}\frac{d}{dy}\left[\frac{\epsilon_m}{\bar{T}}\frac{d\bar{U}}{dy} - \frac{\epsilon_h\bar{U}}{\bar{T}^2}\frac{d\bar{T}}{dy}\right] \quad (6)$$

where we have defined diffusivities in the conventional way by:

$$-\overline{uv} = \epsilon_m\frac{d\bar{U}}{dy}, \quad -\overline{\theta v} = \epsilon_h\frac{d\bar{T}}{dy} \quad (7)$$

The last term in equation (6) gives the contribution of turbulence to the shear stress and the last part of this term represents the extra effect of the density gradient associated with heat transfer. If, as usual, we neglect the molecular contribution to shear in the core of the channel, the surface of zero shear occurs where

$$\epsilon_m\frac{d\bar{U}}{dy} - \epsilon_h\frac{\bar{U}}{\bar{T}}\frac{d\bar{T}}{dy} = 0 \quad (8)$$

The practical use of this condition requires that $\bar{U}$, $\bar{T}$, ϵ_m/ϵ_h be known. However, if we make the approximation $\epsilon_m = \epsilon_h$, equation (8) reduces to the very simple form:

$$\frac{d}{dy}(\bar{U}/\bar{T}) = 0 \quad (9)$$

This equation can also be derived from semi-empirical mixing theories, but the nature of the assumptions made is not so easy to interpret as in the present derivation.

According to equation (9) the position of zero shear occurs where $d/dy\ (\bar{U}/\bar{T}) = 0$ and not where $d\bar{U}/dy = 0$ as is usually assumed. In most practical applications of the zero shear surface technique, $\bar{T} \approx 500°K$. Typical temperature differences between the gas and heated surface seldom exceed a few tens of degrees centigrade, and most of this difference occurs near the surface. Thus, over the region where dU/dy is small, T normally varies much less than 1 per cent. The resulting change in the position of zero shear might well not be significant compared with the experimental errors involved.

REFERENCE

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